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SPATIAL SUPERSONIC MOTION OF A BODY THROUGH A LARGE-SCALE INHOMOGENEITY IN A STRATIFIED ATMOSPHERE†

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The free spatial motion of a blunted body of revolution through a large cloud of heated gas (a thermal) floating in a stratified atmosphere is investigated. Extended composite bodies of the sphere-cone type are considered, the free supersonic motion of which is usually associated with small angles of attack. To determine the trajectory of the body and its orientation in space, a highly effective numerical method is proposed which is an extension of the spatial case of a previously described method [1] for the two-dimensional formulation and which is based on the simultaneous solution of the problems of the flow around a body and its motion. Using this method the effect of a floating thermal on the trajectory and orientation of the body in space and the stability of the flight is investigated in the three-dimensional formulation. The change in the trajectory of the body and its orientation in space as a function of the position of the centre of mass is considered. It is shown that the presence of a thermal may lead to a considerable change in the trajectory of the body and to a loss in flight stability. Copyright © 1996 Elsevier Science Ltd.

A similar problem was considered previously [2] assuming that the trajectory of motion of the body lies in the plane of symmetry of the thermal.

1. FORMULATION OF THE PROBLEM

We will assume that, at the initial instant of time, a cloud of heated gas was formed in a stratified atmosphere of the Earth with the following parameters

$$T(h,r)=T_a(h,r)+(T_{\max}-T_a(h))\exp(-(lR_T^{-1})^2)$$

where $R_T = 4.8$ km, $T_{\max} = 104$ K, $T_a(h)$ is the temperature of the undisturbed atmosphere at an altitude h, and l is the distance from the centre of the spherical volume. The position of the temperature maximum corresponds to an altitude of $H = 20$ km.

The cloud floats under the action of the Archimedes force, forming a vortex ring. The formation of the vortex is accompanied by intense turbulent mixing of the cold and hot layers of air.

Fifteen seconds after the cloud begins to float a body in the shape of a truncated cone on a sphere with a blunt radius $R_0 = 0.1$ m, a semiaperture angle $\zeta = 15°$, a length $L = 2$ m and a mass of 1 t enters the cloud. It is assumed that the centre of mass of the body is at a distance L_c from the vertex. During the initial period of time (when the thermal no longer affects the body motion) the plane of the trajectory is at a sighting distance of 500 m from the axis of symmetry of the thermal, which is situated in the first half-space with respect to the direction of the body motion. The instant when the body is at a distance of 6000 m from the axis of symmetry of the thermal will henceforth be taken as the origin. At this instant the body is at an altitude of 20,000 m and moves horizontally with a velocity of 2000 m/s. The angle of attack at the initial instant is zero, which corresponds to the situation where the pitch angle and the angle of inclination of the body's trajectory are equal, and the yaw and course angles are also equal. We will consider two values of $L_c = 50$ cm and $L_c = 140$ cm, representing the cases of large and small margins of stability of the body, respectively.

2. METHOD OF SOLUTION

A numerical method of calculating the convective–diffusion air flow in the region of a thermal is described in [3]. The motion of a blunt body with a specified rectilinear trajectory is also described, as well as the particular features of the gas flow in the shock layer around the body.

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The trajectory of motion of the centre of mass of the body is given by the following system of equations [4]

$$\dot r = v\sin\theta,\ \ \dot\varphi = \frac{v}{r}\cos\theta\sin\psi,\ \ \dot\lambda = \frac{v}{r}\cos\theta\frac{\cos\psi}{\cos\varphi}$$
$$m\dot v = F_\tau + mr\omega_3^2(\cos\varphi\sin\theta - \sin\varphi\sin\psi\cos\theta)\cos\varphi$$
$$mv\dot\theta = F_n + \frac{mv^2}{r}\cos\theta + 2mv\omega_3\cos\varphi\cos\psi +$$
$$+ mr\omega_3^2(\sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi)\cos\varphi \tag{2.1}$$
$$mv\dot\psi\cos\theta = F_k - \frac{mv^2}{r}\mathrm{tg}\,\varphi\cos^2\theta\cos\psi -$$
$$-2mv\omega_3(\sin\varphi\cos\theta - \cos\varphi\sin\theta\sin\psi) - mr\omega_3^2\sin\varphi\cos\varphi\cos\psi$$

The system of equations (2.1) is written in a geocentric non-inertial spherical system of coordinates, rigidly connected to the Earth; the centre of the system of coordinates coincides with the centre of the Earth. Here **r** is the radius vector of the centre of mass, $r = |\mathbf{r}|$, φ, λ are the geographical latitude and longitude, respectively, v is the modulus of the vector **v** of the velocity of the mass centre, θ is the angle of inclination of the trajectory, ψ is the course angle, m is the mass of the body, ω_3 is the angular velocity of rotation of the Earth, the right orthonormalized vector triple ($\boldsymbol{\tau}$, **n**, **k**) is the accompanying basis of the trajectory of the mass centre of the body ($\boldsymbol{\tau}$ is the tangential unit vector, $\mathbf{v} = v\cdot\boldsymbol{\tau}$, **n** is the normal unit vector and **k** is the unit vector of the binormal to the trajectory), and F_τ, F_n, F_k are the components of the main vector **F** of the forces acting on the body in the associated basis. The initial values of the quantities φ, λ, θ and ψ are assumed to be zero.

The main vector of the forces **F** will be represented in the form $\mathbf{F} = \mathbf{F}^a + \mathbf{F}^g$, where the first term is the resultant of the aerodynamic forces acting on the body, and the second term is the gravitational force.

The orientation of the body in space is governed by Euler's system of equations of motion of a body [5].

We will assume the distribution of the body mass to be axisymmetrical. We will also assume that the angular velocity of rotation of the body around its own axis of symmetry is sufficiently small so that we can neglect the aerodynamic moments of a viscous nature parallel to the axis of symmetry of the body. This assumption reduces to the requirement that the angular velocity of natural rotation of the body is small at the initial instant. In the calculations, the results of which are given below, the angular velocity of rotation of the body at the initial instant is taken to be zero for simplicity.

The gas motions about the body were described by the complete system of equations of a spatial viscous shock layer (see, for example, [6, 7]). To solve this system of equations we used the small-parameter method [7], the small parameter here being the angle of attack, in conjunction with the global iteration method [6, 7].

We thereby obtain the following: the drag $C_x = C_x^{(0)}$, the lifting force $C_y = \alpha C_y^{(1)}$ and the aerodynamic moment $M_z = \alpha M_z^{(1)}$. Here $C_x^{(0)}$, $C_y^{(1)}$ and $M_z^{(1)}$ do not depend on the value of the angle of attack α.

In the spatial formulation of the problem the angle of attack is a three-dimensional vector $\boldsymbol{\alpha}$, which is governed by the vector velocity of motion of the centre of mass of the body with respect to the medium **V** and the unit vector of the axis of symmetry of the body **l**, directed from the centre of mass of the body to the vertex. The modulus of the spatial angle of attack is

$$\alpha = \arccos(\mathbf{V}V^{-1}, \mathbf{l}),$$

while the direction is defined by the vector [**V**, **l**]. Hence, the spatial angle of attack is

$$\boldsymbol{\alpha} = \frac{[\mathbf{V},\mathbf{l}]}{|[\mathbf{V},\mathbf{l}]|}\arccos(\mathbf{V}V^{-1}, l) \approx V^{-1}[\mathbf{V},\mathbf{l}]$$

(since the angle of attack is small).

The total aerodynamic force $\mathbf{F}^a$ acting on the body is the sum of the drag $\mathbf{F}^r$ and the lift $\mathbf{F}^l$, where

$$\mathbf{F}^r = -\frac{\rho V^2 S}{2} C_x \mathbf{l}, \quad \mathbf{F}^l = \frac{\rho V^2 S}{2} C_y^{(1)} [\boldsymbol{\alpha}, \mathbf{l}]$$

(ρ is the density of the free stream and S is the cross-sectional area of the body at its base).

The total moment $\mathbf{M}$ of the forces acting on the body is the sum of the aerodynamic moment $\mathbf{M}^a$, related to the orientation of the body with respect to the free stream, and the damping moment $\mathbf{M}^d$, which occurs due to the rotational motion of the body with respect to the medium.

The aerodynamic moment of the forces about the centre of mass of the body is

$$\mathbf{M}^a = \frac{1}{2} \rho V^2 SLM_z^{(1)} \boldsymbol{\alpha}$$

When $M_z^{(l)}$ has a positive sign the moment of the forces is in the same direction as the angle of attack, which leads to an increase in the latter. The flight in this case is unstable.

The damping moment

$$\mathbf{M}^d = \frac{1}{2} \rho VSL^2 M_\Omega \boldsymbol{\Omega}_\perp$$

where M_Ω is the damping moment coefficient and $\boldsymbol{\Omega}_\perp$ is the component of the angular velocity of rotation vector of the body $\boldsymbol{\Omega}$, orthogonal to the axis of symmetry of the body. (The effect of the rotation of the body around its own axis of symmetry on the gas motion is neglected.) To take the damping moment into account in the general case one must solve the unsteady flow problem. In this paper the coefficient M_Ω was found using Newton's theory [8], which gives acceptable accuracy for Mach numbers of the free stream $M_\infty = 3$. The characteristic ratio M^d/M^a was of the order of 10^{-2}–10^{-3} in the calculations, so that the damping moment could be neglected in practice in our formulation of the problem.

The whole system of governing equations was solved as follows.

We write the system of equations of the body motion in the form

$$\frac{d}{dt} \boldsymbol{\xi} = \mathbf{F}(\boldsymbol{\xi}, \mathbf{A}, \mathbf{Q}), \quad \mathbf{A} = \left\| C_x^{(0)} C_y^{(1)} M_z^{(1)} \right\|^T \tag{2.2}$$

Here $\boldsymbol{\xi}$ is the required vector which completely describes the kinematic characteristics of the body motion (the coordinates, velocities and angles); the vector $\mathbf{Q}$ has its own components, which are required to calculate the parameters of the thermal (the horizontal and vertical components of the flow velocity in the thermal, and the density and temperature of the air at the point of space and the instant of time considered), and $\mathbf{A}$ is the vector of the aerodynamic coefficients, where $\mathbf{A} = \mathbf{A}(\boldsymbol{\xi}, \mathbf{Q})$, $\mathbf{Q} = \mathbf{Q}(\boldsymbol{\xi}, t)$. The vector $\mathbf{Q}$ depends on time due to the gas motion in the thermal.

At the initial instant the system of equations of the viscous shock layer is solved for specified free stream parameters. Then, during a time Δt_g the system of ballistic equations is integrated for a specified position and velocity of the centre of mass, and also the body orientation in space with fixed values of C_x, $C_y^{(1)}$ and $M_z^{(1)}$ and variable angle of attack α. The step Δt_g is determined by the characteristic time of the changes in the free stream parameters, which considerably exceeds the characteristic time of variation of the angle of attack (the body undergoes oscillations about the position corresponding to $\alpha = 0$), as a consequence of which the integration step of the system of ballistic equations $\Delta t_b \ll \Delta t_g$.

Consider system (2.2) in the time interval $[t^n, t^{n+1}]$ where $t^{n+1} = t^n + (\Delta t_g)^n$. We will assume that at the instant t^n we know the values and derivatives with respect to time of the vectors $\boldsymbol{\xi}$ and $\mathbf{Q}$, obtained when system (2.2) is integrated during the preceding time interval $[t^{n-1}, t^n]$, and also the values $\mathbf{A}^{n-1}$ and $\mathbf{A}^n$ of the vector $\mathbf{A}$ at the instants of time t^{n-1} and t^n. We put $\boldsymbol{\xi}^n = \boldsymbol{\xi}(t^n)$.

The solution of system (2.2) in the interval $[t^n, t^{n+1}]$ is found using the "predictor–corrector" scheme.

At the "predictor" stage we integrate the system

$$\frac{d}{dt} \boldsymbol{\xi}_0 = \mathbf{F}(\boldsymbol{\xi}_0, \mathbf{U}_0^n(t), \mathbf{G}^n(t)), \quad \boldsymbol{\xi}_0(t^n) = \boldsymbol{\xi}^n \tag{2.3}$$

where

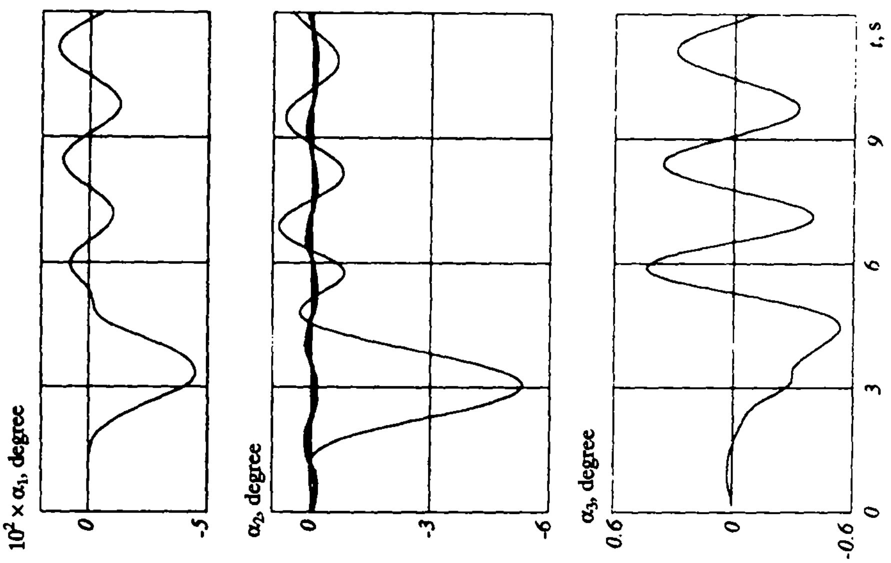
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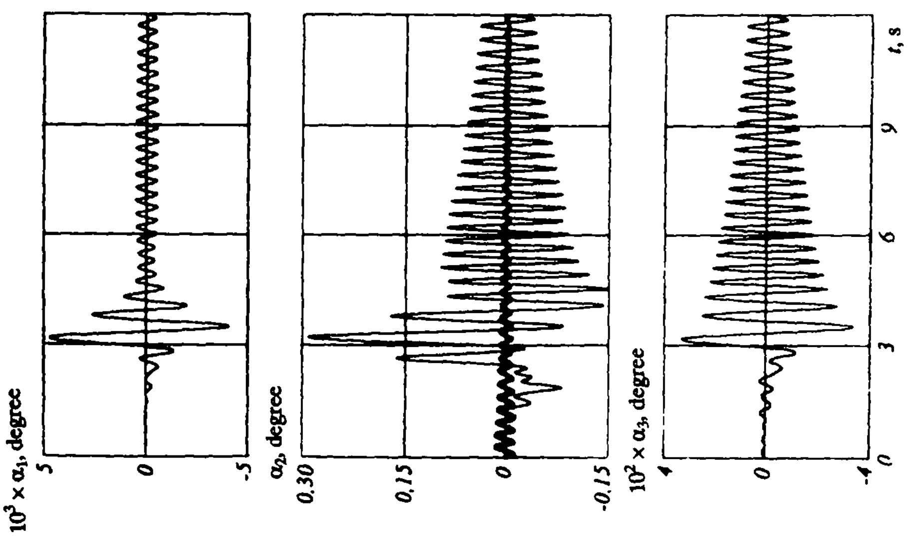
Fig. 2.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-4
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9
t, s

Fig. 1

$$U_0^n(t) = A^n + \frac{A^n - A^{n-1}}{(\Delta t_g)^{n-1}}(t - t^n)$$

and $G^n(t)$ is the local spline, which approximates the function $Q(t)$ in the interval $[t^n, t^{n+1}]$, constructed from the values of the function $Q(t)$ and its derivatives at the points t^n and t^{n+1}.

At the "corrector" stage we integrate the system

$$\frac{d}{dt}\xi = F(\xi, U^n(t), G^n(t)), \quad \xi(t^n) = \xi^n \tag{2.4}$$

where

$$U^n(t) = A^n + \frac{A_0^{n+1} - A^n}{(\Delta t_g)^n}(t - t^n)$$

$$A_0^{n+1} = A(\xi_0^{n+1}, Q(\xi_0^{n+1}, t^{n+1}))$$

ξ_0^{n+1} is the value of the solution of system (2.3) when $t = t^{n+1}$, and A_0^{n+1} is the value of the vector A at the instant $t = t^{n+1}$, which is obtained from the solution of the problem of flow with free stream parameters defined by the vector ξ_0^{n+1}. In view of the closeness of the values of $A^{n+1} = A(\xi(t^{n+1}), Q(\xi(t^{n+1}), t^{n+1}))$ and A_0^{n+1}, to solve the problem in the time interval $[t^{n+1}, t^{n+2}]$ we assume $A^{n+1} = A_0^{n+1}$.

Systems (2.3) and (2.4) are solved by the method of successive approximations, which is realized after 2–4 iterations with an error of less than 1%. Here, we take as the initial approximation for $G^n(t)$ at the "predictor" stage the function

$$G_0^n(t) = Q(t^n) + \frac{d}{dt}Q(t^n)(t - t^n) + \frac{d^2}{dt^2}Q(t^n)(t - t^n)^2$$

Integration of system (2.4) with an accuracy of $O(\Delta t_g^2)$ gives a solution of the initial system (2.2).

At each iteration of the method of successive approximations, the system of equations (2.3) and (2.4) was integrated by a Runge–Kutta method of third order of accuracy in Δt_b. Note that system (2.2) is hard, since the characteristic times of variation of the required quantities in the equations, from the definition of the body orientation, is much less than in (2.1). However, at each iteration of the Runge–Kutta method, after linearizing the moment M of the forces and the components of the vector Ω on the right-hand sides of Euler's equations of body motion, the latter can easily be integrated analytically, which considerably reduces the requirements imposed on the integration step Δt_b.

In view of the fact that, in each subsequent solution of the equations of the spatial viscous shock layer (over a time interval Δt_g), we used a good initial approximation from the preceding solution, 3–5 global iterations in all were required for convergence.

The characteristic ratio $\Delta t_b/\Delta t_g$ was 10^{-2}–10^{-3}. The time step Δt_g was assigned values of 0.2–3 s depending on the rate of variation of the free stream parameters along the flight trajectory.

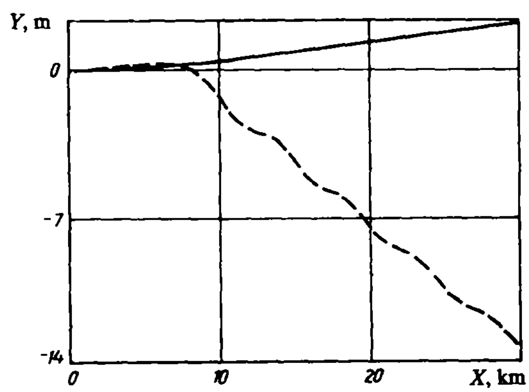

Fig. 3.

A typical time taken to calculate the interval of physical time Δt_g was 35–40 min on an IBM 386/387 computer.

3. RESULTS

For the values of L_c considered, the centre of pressure in the unperturbed atmosphere is behind the centre of mass, and the body flight is statistically stable. When the body moves to the centre of the thermal the density of the free stream falls rapidly while the temperature rises, and this leads to a reduction in both the Mach number and the Reynolds number Re_∞.

The reduction in M_∞ for a fixed value of Re_∞ should lead to a displacement of the centre of pressure towards the cone vertex. At the same time, as shown in [2], a reduction in Re_∞ for fixed M_∞ causes a shift in the centre of pressure in the opposite direction. Nevertheless, the overall effect of a change in both M_∞ and Re_∞ along the trajectory leads to a displacement of the centre of pressure towards the cone vertex, which reduces its stability. Here, in the neighbourhood of the centre of the thermal, the centre of pressure is ahead of the centre of mass for L_c = 140 cm and the body flight becomes statistically unstable. This may lead to inversion of the body [2].

During its motion the body may undergo oscillations about the position corresponding to zero angle of attack. Here the amplitude of the oscillations of the angle of attack is determined by the margin of stability of the body, which can be seen from Figs 1 and 2, where we show, for L_c = 50 cm and L_c = 140 cm, respectively, the projections α_1, α_2, α_3 of the vector of the angle of attack $\boldsymbol{\alpha}$ onto the axis of a local system of coordinates with basis $(\mathbf{r}_\lambda | \mathbf{r}_\lambda|^{-1}, \mathbf{r}_\varphi | \mathbf{r}_\varphi|^{-1}, \mathbf{r}r^{-1})$ as a function of the time of flight. (The thick curves represent graphs corresponding to the motion in an undisturbed atmosphere.) In the second version, which represents the case of a small margin of stability, the body is oriented extremely slowly in the direction of the free stream. Excitation of the oscillations of the components α_1 and α_3 is due to the body entering the region of the thermal along a trajectory that does not lie in its plane of symmetry and is the reason for the change in the initial course and the side drift of the body.

The behaviour of the pitch of the body as it moves through the thermal, and also the change in the trajectory of motion in the vertical plane (the dependence of the altitude of flight on the distance) is not much different from the case of motion in the plane of symmetry of the thermal, considered in [2].

When modelling the spatial motion of a body through a thermal an interesting effect was obtained, arising from the possible strong deviation of the body from its course. This effect is also due to a shift in the centre of pressure when it enters the region of heated gas. In Fig. 3 we show projections of the trajectory of the body onto the horizontal plane. The X axis is directed along the projection of the velocity vector onto the plane of the local horizon at the initial instant of time, the Z axis is determined by the

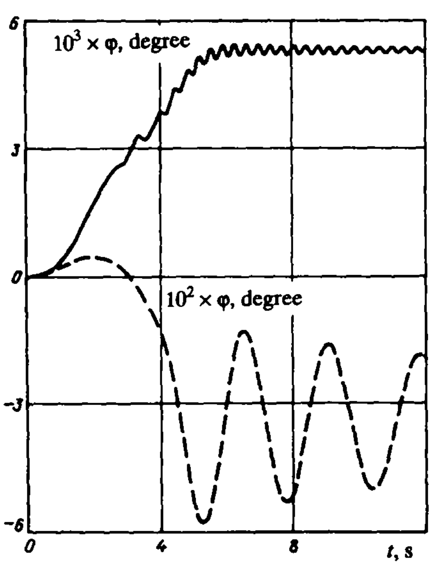

Fig. 4.

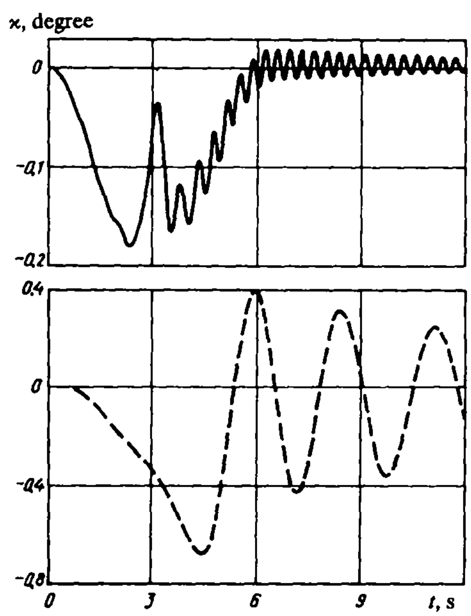

Fig. 5.

vector **r** at the initial instant, and the Y axis is orthogonal to the X axis, so that the unit vectors of X, Y and Z axes form a right triple. When the body moves through a thermal the "wind", due to the flow in the region of the thermal, has only a positive component on the Y axis. When $L_c = 50$ cm the body, correspondingly, is easily deflected to the left of the direction of the "wind" by the free stream (the continuous curve in Fig. 3).

Another situation occurs when $L_c = 140$ cm. Here the body, on moving through the thermal, is deviated to the right in the opposite direction of the free stream (the dashed curve in Fig. 3). This, at first sight, paradoxical result can be explained as follows. Due to the shift of the centre of pressure and its closeness to the centre of mass, the body becomes inert to rotating loads. As a result, the turning of the body along the flow becomes much slower than in the first version.

In Fig. 4 we show graphs of the course angle ψ as a function of time, while in Fig. 5 we show graphs of the yaw $\varkappa$ (the angle between the projections of the vector of the axis of symmetry of the body onto the plane of the local horizon and the local parallel) as the function of the time of flight. (The continuous curve corresponds to the case when $L_c = 50$ cm, and the dashed curve corresponds to the case when $L_c = 140$ cm.)

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